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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
Nov 1998

3. REPORT TYPE AND DATES COVERED
Final 01 May 95 – 30 Apr 98

4. TITLE AND SUBTITLE
Controlled Diffusion Approximations for Controlled Queueing Systems

5. FUNDING NUMBERS
DAAH04-95-1-0226

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 33014.3-MA

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)
Four problem areas were studied. These are (1) controlled heavy traffic queueing systems, (2) queueing systems with due dates, (3) backward-forward stochastic differential equations, and (4) Ginzburg-Landau equations and evolving interfaces. In areas (1) and (2), diffusion approximations were obtained for queues in heavy traffic. In (3), connections were established between quasi-linear partial differential equations and diffusion processes constructed via a new class of stochastic differential equations. Finally, (4) provides a study of the partial differential equation characterizing vortices in superconducting material in three dimensions.

14. SUBJECT TERMS
Diffusion approximations, queueing systems, Ginzburg-Landau equations, quasi-linear partial differential equations

15. NUMBER OF PAGES
3

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

FINAL REPORT: DAAH04-95-1-0226

May 1, 1995 – April 30, 1998

Title: Controlled Diffusion Approximations for Controlled Queueing Systems

ABSTRACT

Four problem areas were studied. These are (1) controlled heavy traffic queueing systems, (2) queueing systems with due dates, (3) backward-forward stochastic differential equations, and (4) Ginzburg-Landau equations and evolving interfaces. In areas (1) and (2), diffusion approximations were obtained for queues in heavy traffic. In (3), connections were established between quasi-linear partial differential equations and diffusion processes constructed via a new class of stochastic differential equations. Finally, (4) provides a study of the partial differential equation characterizing vortices in superconducting material in three dimensions.

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During the period of funding, B. Doytchinov earned a Ph.D. in mathematics.

This grant supported work on four different projects:

1. Controlled heavy traffic queueing systems

2. Queueing systems with due dates

3. Backward-forward stochastic differential equations

4. Ginzburg-Landau equations and evolving interfaces.

Queueing systems are highly complex and not amenable to detailed analysis. It is nonetheless desirable to have quantitative measures of the performance

which one can expect from these systems under heavy traffic conditions. A very successful approach to this problem has been to approximate the queueing system by a Brownian network, a special case of a multi-dimensional diffusion process.

While Brownian network approximations are well understood and provide accurate performance evaluation results for uncontrolled queueing systems, the connections between Brownian networks and the underlying queueing systems are much less well understood when the possibility of control of the queue is allowed. The first project was to develop this relationship at a mathematically rigorous level for a representative queueing system. This resulted in (1).

Brownian networks normally model the number of tasks in queue, but not their dynamic attributes. A quantity not modelled previously but of particular interest is the lead-time profile of the set of tasks in queue. One can model these lead times as points on the real line, and the whole lead-time profile becomes a counting measure. Under the heavy traffic scaling, when the earliest-deadline-first service protocol is used, this lead-time profile measure converges to a limit, whose dynamic properties were obtained in (7). An analysis when the first-in-first-out protocol is used is provided by (6).

The classical Feynmann-Kac formula provides a characterization of solutions to second order parabolic, *linear* partial differential equations in terms of diffusion processes. A more complicated formula yields a connection between a certain class of differential equations and controlled Markov processes. This formula is restricted to "convex" type nonlinearities. Slightly more general theory is available if one considers stochastic differential games instead of optimal control.

An exciting new direction in this line of research was the discovery of a similar connection between backward-forward stochastic differential equations and a more general class of quasi-linear partial differential equations. In joint work with Cvitanic and Karatzas, Soner studied the backward-forward stochastic differential equations with *constraints.* The resultying equation is a quasi-variational inequality. This connection and other properties were studied in (5).

In a different direction, Jerrard and Soner studied the asymptotic behavior of Ginzburg-Landau systems. Earlier analytical work has been restricted to scalar equations. In (3) and (4), new techniques in analyzing systems have been developed. In particular, (3) provides a rigorous justification of vortex equations. This equation is the gradient flow with a logarithmic potential and it models the transient behavior of vortices in a superconducting material after it is subjected to a high applied magnetic field. Publication (4) studies the similar situation in three space dimensions. In this case, singularities are no

longer point vortices but vortex lines.

The following publications resulted from this grant:

(1) L. F. Martins, S. E. Shreve and H. M. Soner, Heavy traffic convergence of a controlled, multi-class queueing system *SIAM J. Control and Optimization* **34** (1996) 2133–2171.

(2) R. Jerrard & H. M. Soner, Dynamics of Ginzberg-Landau vortices, *Archive Rational Mechanics Analysis*, **142/2** (1998), 9–125.

(3) R. Jerrard & H. M. Soner, Scaling limits and regularity for a class of Ginzburg-Landau systems, *Annales l'Institute H. Poincare; Analyse Non-linear*, to appear,

(4) K. Ishii & H. M. Soner, Regularity and convergence of crystalline motion, *SIAM J. Mathematical Analysis*, to appear.

(5) J. Cvitanić & H. M. Soner, Backward stochastic differential equations with constraints on the gains process.

(6) B. Doytchinov, Heavy traffic limits of queues with due dates, Ph.D. dissertation, Department of Mathematical Sciences, Carnegie Mellon University, 1997.

(7) B. Doytchinov, J. Lehoczky and S. Shreve, Real-time queues in heavy traffic with earliest-deadline-first queue discipline, submitted for publication.